Memo to All Structural Engineers Conducting Earthquake Analyses

From: Ed Wilson
In his ninetieth year

Introduction

Professor Nathan M. Newmark at the University of Illinois was a pioneering earthquake engineer who used early digital computers to solve many complex structures – including the first generation of many Nuclear Reactors for power generation in the US. In approximately 1959, he presented a seminar to a group of structural engineers at UC Berkeley. Near the end of his lecture, after making many assumptions and mathematical manipulations, he summarized his lecture by indicated - ***the earthquake dynamic analysis of a building is very similar to a wind analysis of the same structure***. As a young graduate student this did not sound physically possible. However, for over 60 years most structural engineers, including myself, have used the same approach as Newmark suggested – since a good alternative to the "Relative Displacement Method" had not been proposed.

In the early nineteen nineties a group of earthquake engineers met at Cal Tech to participate in a ***Symposium in Honor of Professor George Housner*** for his many contributions to the fields of physics and mathematics of earthquake engineering. Near the Symposium as I said goodbye to George, he essentially asked me: ***Do you known, the method***

of dynamic earthquake analysis you used to write all those computer programs, does not satisfy the Fundamental Laws of Conservation of Energy? I replied no – but would check it out. The method he was referring to was the "Relative Displacement Method" which was recommended by Professor Newmark over 30 year previously.

Finally, within the last year, I have:

1. Identified the flawed physics and mathematical assumptions in the development of the "Relative Displacement Method" and
2. Demonstrated the "Total Displacement Method" is a more accurate approximation and satisfies the basic energy, equilibrium, and mathematical laws.
3. Finally,

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Relative Displacement Method

All structures are in a continuous state of dynamic motion because of random loading such as wind, vibrating equipment, or human loads. These small ambient vibrations are normally near the natural frequencies of the structure and are terminated by energy dissipation in the real structure.

Real structures are made of a finite number of elements interconnected at a finite number of joints. For a dynamic earthquake analysis, this real structure must be connected to the earth at the foundation as shown below in Figure 1.

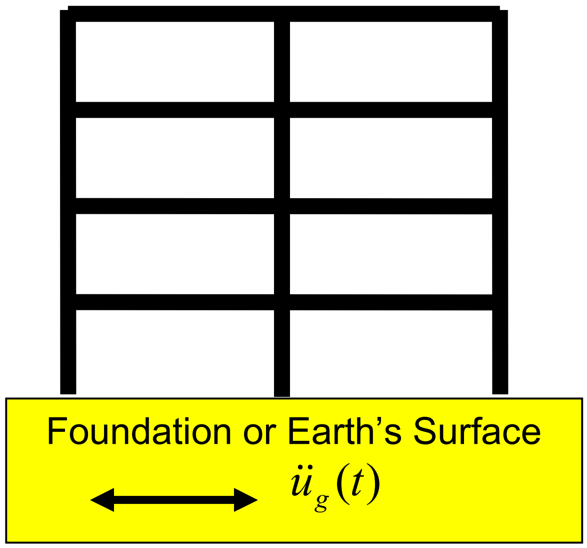

Figure 1. Real Building Structure with a Foundation

Each element in a structure has mass (which has kinetic energy when vibrated) and the ability to deform when subjected to loads (strain energy). Both exist in every particle within every member. The earthquake acceleration, $\ddot{u}_g(t)$, is specified only at the surface of the earth. However, it must be attached to bottom of structure if the earthquake energy can enter the structure.

Now; prior to moving the mass of each element to the end-joints, we must recognize that it is impossible to physically to do this. This is a critical approximation as we progress to the next steps in the development of the ***Relative Displacement Method***. The only computational lumped-mass model used by the developer of the method is shown in Figure 2.

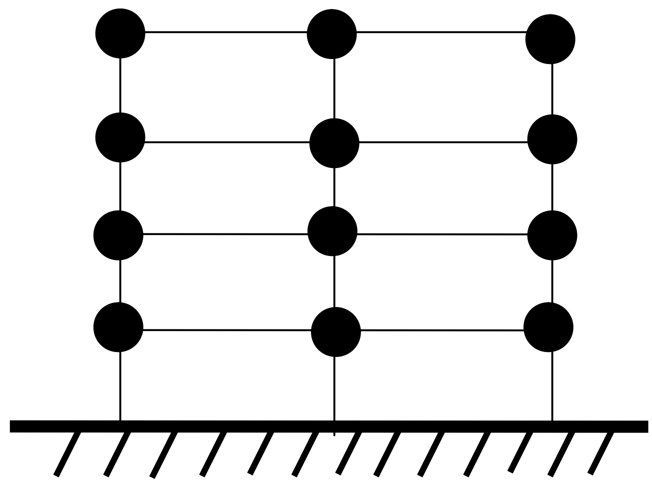

Figure 2: *Lumped Mass – Fixed Base Building Model*

At the foundation level zero displacements are specified; therefore, no other loads or displacements can be specified at the fixed joints. The free-vibration, dynamic equilibrium, equations for the lumped mass model shown in Figure 2 are summarized in Equation (1) - using matrix notation. Where the unknown relative displacements $u_r(t)$ and the relative accelerations $\ddot{u}_r(t)$ are respect to the zero values at the base of the structure. Also, $M_{s,}$ is a diagonal mass matrix and K_s is a sparse stiffness matrix,

$$[M_s \ddot{u}_r(t) + K_s u_r(t)] = 0 \quad (1a)$$

Also, in terms of conservation of energy, Equation (1a) must satisfy the following scalar equation at every point in time "t".

$$1/2\,\dot{u}_r^{\,T} M_s \dot{u}_r + 1/2\,u_r^{\,T} K_s u_r = consant\ value\ or\ 0 \quad (1b)$$

This is the fundamental equation of physics where the sum of kinetic energy and strain energy must exist if vibration, without external loads, can take place. Both of these laws, summarized in Equation (1a and 1b), are ***physics requirements***. Therefore, great care must be used in the execution of any subsequence ***mathematical manipulation*** of these equations. You cannot discard the stiffness without discarding the mass.

The next step in derivation of the ***Relative Displacement Method*** is to assume the total displacement $u(t)$ was equal to the sum of the relative displacement $u_r(t)$ and the displacement $u_b(t)$ at the base of the building which had been set to zero. Or:

$u(t) = u_r(t) + u_b(t)$ or $u_r(t) = u(t) - u_b(t)$ and

$\ddot{u}(t) = \ddot{u}_r(t) + \ddot{u}_b(t)$ or $\ddot{u}_r(t) = \ddot{u}(t) - \ddot{u}_b(t)$

They then assumed $u_r(t)$ could eliminate from the free vibration of Equation (1) and obtain:

$$[M_s\ddot{u}(t) + K_s u(t)] - [M_s\ddot{u}_b(t) + K_s u_b(t)] = 0 \qquad (2)$$

Since the base of the building is still fixed at zero displacement and zero acceleration, Equation (1) has not changed. The building still has not been attached to the foundation where the earthquake energy should enter the building – see Figure 1 and 2. Others, developers have attempted to use D'Alembert's principle to justify setting $K_s u_b(t)$ to zero – this is not possible since K_s is not a rigid body. Also, K_s and

M_s occupy the same space and cannot be separated by discarding one and keeping the other.

The original developers of the ***Relative Displacement Method*** made the mistake of defining $\ddot{u}_b(t)\ as\ \ddot{u}_g(t)$. ***(Error #1)*** Then they moved $M_s\ddot{u}_g(t)$ to the right hand side of Equation (2). In addition, they incorrectly claimed $K_s\ddot{u}_g(t)$ was equal to zero – they neglected to realize the stiffness matrix K_s was for a fixed base building. ***(Error #2)***

These approximations result in an ***erroneous*** equation for the ***Relative Displacement Method***

$$[M_s\ddot{u}(t) + K_s u(t)] = -M_s\ddot{u}_g(t) \tag{3}$$

Equation (3) indicates the earthquake loading is applied to all mass points, within a three-dimensional building, above the foundation. For many short and stiff buildings the method can produce a reasonable approximate result when compared to the results of all the earthquake energy applied at the foundation level as done in real earthquake, shaking table experiments and ***The Total Displacement Method.***

However, Equation 3 is obvious incorrect since there is not a physical path for the earthquake energy from the earth, to propagate through air, to enter the upper stories of a tall building. It is necessary for the earthquake energy waves must enter ***at the foundation contact area between the shaking earth and the base of the building.***

Personally, it is very difficult for me to believe that all the experts in the academic world, who teach earthquake engineering and structural dynamics, agree with the approximations used to derive Equation 3 for the past 60 years. ***This method is a great example of flawed physics and flawed mathematics.***

This year is fifty year since I release the first version of the SAP[1] series of programs which has the relative displacement method as the default option. Approximately twenty years ago I published in my book[2] a theoretically correct method to apply earthquake displacement at the base of a structure. However, it required the calculation of very high frequency mode shapes (static modes) and was not practical.

[1] Structural Analysis Program ---- 1970

[2] Static and Dynamic Analysis of Structures - 2005

Total Displacement Method

This author is pleased to present an old approach for the seismic computer analysis of structures which is based on sound principal of physics. The approach is identical to the one used in all shaking table tests conducted for over the last hundred years. Therefore, it is not necessary to make additional assumptions. In fact, the relative displacement method was developed in the mid fifties - in order to use the newly created digital computers to produce a seismic analysis. After over 60 years, it is time to compare the two methods with a simple building example shown in Figure 3.

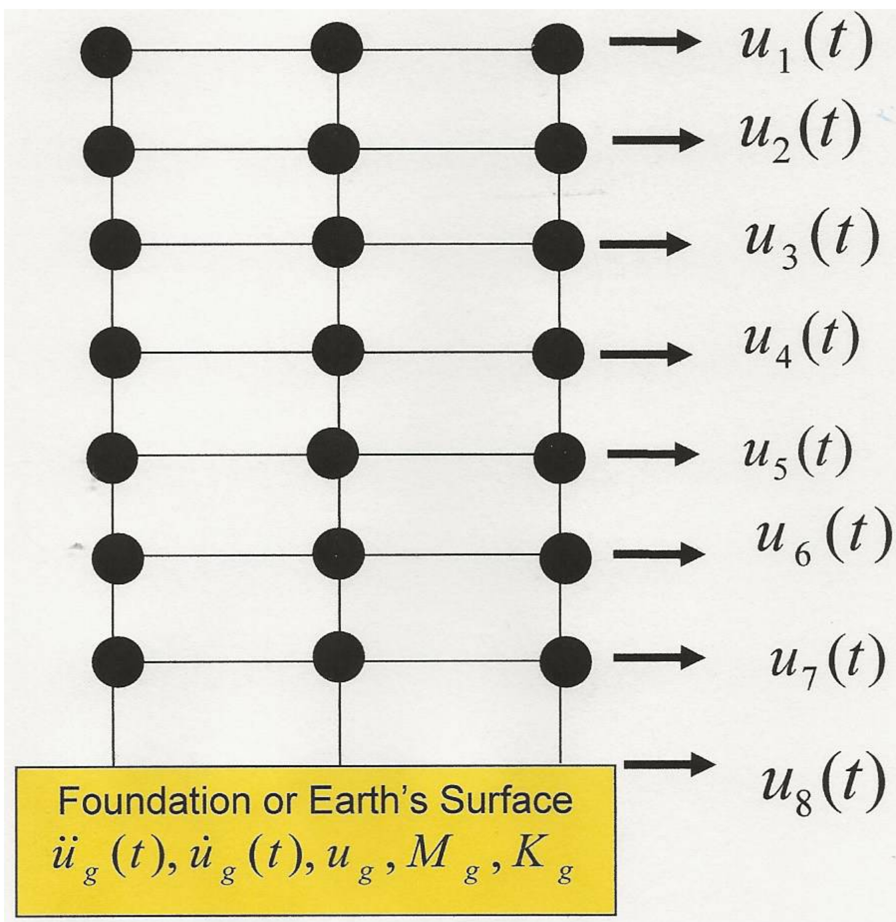

Figure 3: *Simple building model with and without foundation*

This simple example structure is a seven story (13 ft each) by two bay (20 ft each) building. The seven floor girders are ridged with respect to both axial and bending deformations. The columns, between the girders, are rigid in the axial direction and their total floor-to-floor bending stiffness are the sum of the $12EI/h^3$, or, 150 ***k /in*** per story. The total mass of the columns, girders and floor slab is set at 0.20 ***k sec² /in*** per story.

The dynamic equilibrium equations, in terms of the unknown total displacements $u_s(t)$ within the superstructure, and the foundation displacements $u_g(t)$ in sub matrix form is:

$$\begin{bmatrix} M_s & \mathbf{0} \\ \mathbf{0} & M_g \end{bmatrix} \begin{bmatrix} \ddot{u}_s(t) \\ \ddot{u}_g(t) \end{bmatrix} + \begin{bmatrix} K_s & K_{sg} \\ K_{gs} & K_g \end{bmatrix} \begin{bmatrix} u_s(t) \\ u_g(t) \end{bmatrix} = \begin{bmatrix} 0 \\ F_e(t) \end{bmatrix} \quad (4)$$

The solution of this equation may be a challenge; since, we do know the values of the foundation mass, $M_g(t)$, and foundation stiffness, $K_g(t)$. However, we know most of the laws of physics and we know the free-field earthquake accelerations prior to the construction of the building. Therefore, I am confident we can improve on the assumptions used to create the "Relative Displacement Method" where $M_g(t)$ was set to zero and $K_g(t)$ was arbitrarily set to infinity (to fix the base).

In order to select a realistic value of the foundation mass, let us evaluate the free vibration frequencies and mode shapes for both models.

Table 1 Comparison of frequencies vs. foundation masses

Fixed base Model Total Mass =1.40 Kg = 150,000		Super Structure on Floating Foundation		
		Mass = 1.40 Kg = 150	Mass = 14.0 Kg = 150	Mass = 140 Kg = 150
Number	Period Sec	Period Sec	Period Sec	Period Sec
1	1.099	8.624	20.15	61.06
2	0.372	0,829	1.0545	1.094
3	0.230	0.355	0.370	0.371
4	0.172	0.226	0.229	0.229
5	0.142	0.170	0.172	0.172
6	0.126	0.141	0.142	0.142
7	0.117	0,126	0.126	0.126
8	None	0.117	0117	0.117

Table 1 indicates, as the mass of the foundation is increased, while holding its stiffness constant, the foundation mode approaches a ridge-body mode and the building frequencies approach those of the fixed base model. Therefore, the foundation mode represents the motion of the top of a concrete shaking table with large hydraulic jacks supplying the earthquake energy to the structure.

END OF DAY: Oct. 21. 2020

Comparison between the Relative and Total Displacement Methods

A steady-state harmonic, earthquake acceleration $\ddot{u}_e(t)$ is applied to the foundation mass; the solution to the differential equation of Equation 4 is reduced to a Static load problem.

where λ is the frequency (in radians per second) of the harmonic load.

$u_s(t) = U_s Sin(\lambda t)$ and $\ddot{u}_s(t) = -U_g \lambda^2 Sin(\lambda t)$

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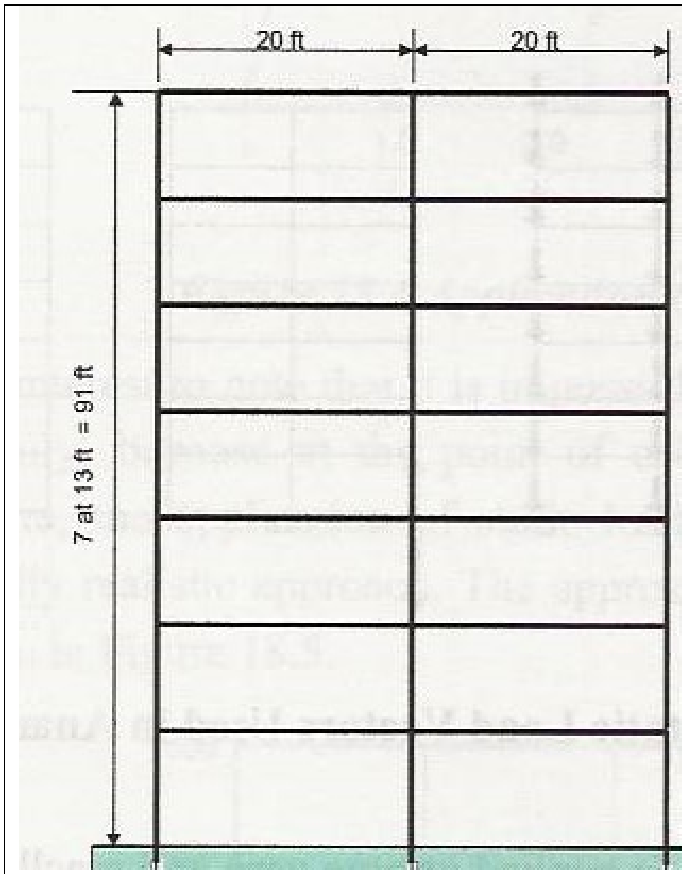

Member Properties

Beams ***Rigid***
Columns $I=1000\ in^4$
Modulus ***E=5,000 ksi***
Story height ***h=13 ft***
=157 in
Lateral Stiff. $12EI/h^3$
= 46 kips/in

weight per story =500 x 40 =20 kips

mass =W/g =20/386 =k/in/sec2

m = 0.20 k sec^2 /in per story

The total displacement method is the correct physical and mathematical approach to solve the problem. The current relative displacement method has proven is be *erroneous* and should be discontinued ***as soon as an alternative method is made available to the profession,*** immediately. Therefore, if you are a careful and a creative modeler you will be able to do real dynamic soil-structure problems

The dynamic equilibrium of the building and foundation, as defined by Equation 4, can be solved by using the standard mode superposition method for the total displacements and element forces as a function of time. Friction energy dissipation can be added when the modal equations are solved for each mode.

The foundation has mass M_g only and the earthquake forces are calculated from $F_g(t) = M_g \ddot{u}_g(t)$. Therefore, the foundation has extra rigid body modes which will also move the superstructure when activated. Guidelines for calculating M_g and K_g will be given when we present a comparison between the relative and total displacement methods.